Explicit Realization of Chaos Control in an NMR-Laser Experiment

J. Parisi [1], R. Badii [2], E. Brun [3], L. Flepp [3], C. Reyl [3], R. Stoop [4], O. E. Rössler [5], A. Kittel [6], and R. Richter [6]

[1] Physical Institute, University of Bayreuth, Bayreuth, Germany
[2] Paul Scherrer Institute, Villigen, Switzerland
[3] Physics Institute, University of Zurich, Zurich, Switzerland
[4] Institute of Mathematics, ETH Zurich, Zurich, Switzerland
[5] Institute for Physical und Theoretical Chemistry, University of Tübingen, Tübingen, Germany
[6] Physical Institute, University of Tübingen, Tübingen, Germany

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The usefulness of the Ott-Grebogi-Yorke control method is demonstrated by stabilizing a chaotic NMR-laser system around an unstable period-one orbit. We have used a six-dimensional delay-coordinate embedding technique in order to fully determine the stability properties of the orbit controlled. Our analysis yields small time-dependent perturbations of the system quality factor capable to perform real-time control.

A nonlinear dynamical system may exhibit chaotic spatio-temporal behavior as a suitable control parameter is varied. Elimination of aperiodicity is often desired. The difficulty of the task is clearly related to the complexity of the dynamics. Ott, Grebogi, and Yorke [1] first had the idea to take advantage of the properties of a chaotic system, i.e., the sensitivity to small perturbations and the ergodicity of the motion, in order to control the dynamics. Here, control means to sway the system so as to bring it in a reproducible, predictable state. This can be achieved by an intelligent feedback technique in which small, carefully chosen, time-dependent perturbations are applied to one of the control parameters.

We start from the assumption that a chaotic attractor is interspersed with an infinite number of dense unstable periodic orbits, providing a convenient reservoir of possible trajectories to stabilize on. The goal was not to force the system to follow some arbitrary periodic perturbation, but to hold the chaotic motion close to an already existing unstable periodic orbit. This can be realized by confining the trajectory on the stable, attracting manifold of the chosen unstable orbit by means of repeated intelligent perturbations. We have implemented that method to a parametrically modulated nuclear magnetic resonance (NMR) laser experiment [2] and proceeded as follows.

First, the unstable periodic orbits are extracted from a chaotic time series embedded in a six-dimensional phase space [3, 4]. Second, we determine the local unstable and stable manifolds at some point (i.e., the control station) on a given unstable periodic orbit [5]. Third, we systematically investigate the influence of the control parameter (i.e., the quality factor of the laser system) upon the position of the control station in phase space. Finally, the real-time control procedure takes place: the state of the laser is reconstructed and its analysis yields the magnitude of the perturbation which is directly applied to the control parameter.

Figures 1 and 2 show first results of successfully stabilizing laser chaos onto the period-one orbit with perturbations of magnitude no larger than four percent of the absolute value of the control parameter. The control has been reproduced many times and tested during long-term operation (more than one hour). It even resisted strong external disturbances, and outbursts were observed no longer than fractions of a second. A more detailed treatment is the topic of a forthcoming publication.

We have demonstrated the validity of a dynamical control method by means of weak perturbations which do not alter the nature of the system considered. There are several remarkable features: (i) Stabilization of chaos with respect to one out of many different coexisting unstable periodic orbits is definitely possi-

Reprint requests to Prof. Dr. J. Parisi, Physical Institute, University of Bayreuth, Box 10 12 51, W-8580 Bayreuth, Germany; FAX + +49-921-552621.

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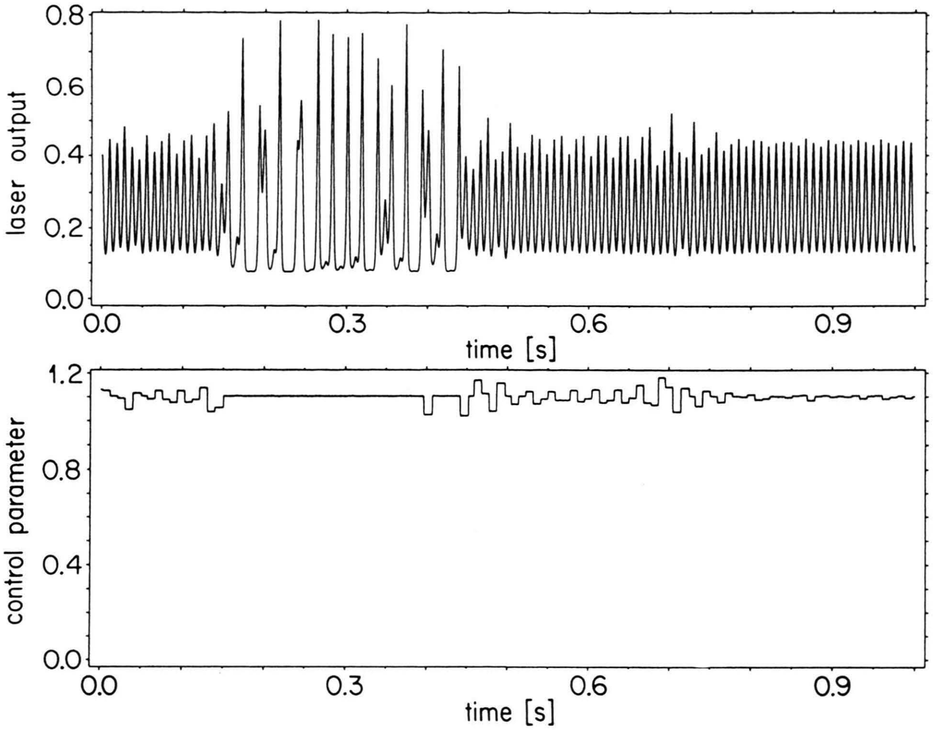

Fig. 1. Temporal structure of the laser output (upper trace) partly stabilized to a period-one oscillation by the help of exerting specific time-dependent corrections on the control parameter (lower trace). The erratic outburst in the laser signal clearly derives from an intermittent loss of control (see absence of correction pulses).

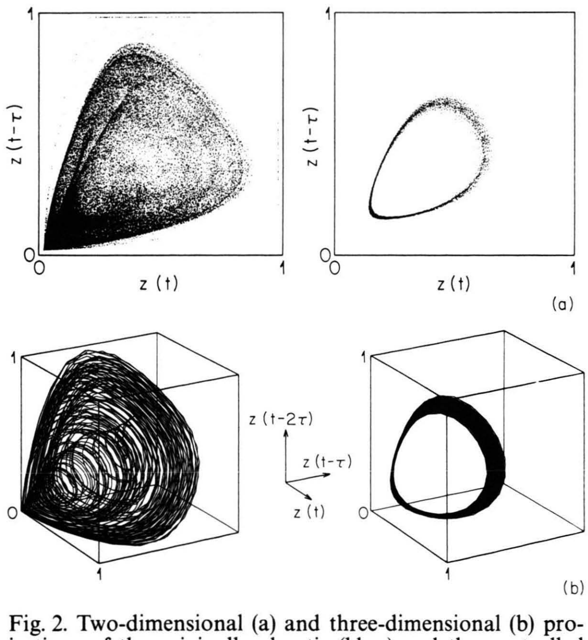

Fig. 2. Two-dimensional (a) and three-dimensional (b) projections of the originally chaotic (l.h.s.) and the controlled period-one (r.h.s.) orbit, each time constructed from a time series of laser data (taking about four seconds) via the usual time-delay method.

ble. (ii) No dynamical equations describing the underlying physics are needed. (iii) The computational effort is small, opening the door to various real-time applications. Control of many complex chaotic systems may be practicable in the future.

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